SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Determine the possible values of the random variable.

1) Suppose that two balanced dice are rolled. Let X denote the absolute value of the difference of the two numbers. What are the possible values of the random variable X?

2) For a randomly selected student in a particular high school, let Y denote the number of living grandparents of the student. What are the possible values of the random variable Y?

Use random-variable notation to represent the event.

3) Suppose a coin is tossed four times. Let X denote the total number of tails obtained in the four tosses. Use random-variable notation to represent the event that the total number of tails is three.

4) Suppose that two balanced dice are rolled. Let Y denote the sum of the two numbers. Use random-variable notation to represent the event that the sum of the two numbers is at least 3 but less than 5.

Obtain the probability distribution of the random variable.

5) When two balanced dice are rolled, 36 equally likely outcomes are possible as shown below.

\[
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6) \\
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6) \\
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6) \\
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6) \\
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6) \\
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)
\]

Let X denote the absolute value of the difference of the two numbers. Find the probability distribution of X. Give the probabilities as decimals rounded to three decimal places.

Construct the requested histogram.

6) If a fair coin is tossed 4 times, there are 16 possible sequences of heads (H) and tails (T). Suppose the random variable X represents the number of heads in a sequence. Construct the probability distribution for X.

Find the specified probability.

7) A statistics professor has office hours from 9:00 am to 10:00 am each day. The number of students waiting to see the professor is a random variable, X, with the distribution shown in the table.

\[
\begin{array}{c|cccccc}
 x & 0 & 1 & 2 & 3 & 4 & 5 \\
 P(X = x) & 0.05 & 0.10 & 0.40 & 0.25 & 0.15 & 0.05 \\
\end{array}
\]

The professor gives each student 10 minutes. Determine the probability that a student arriving just after 9:00 am will have to wait no longer than 20 minutes to see the professor.
Provide an appropriate response.

8) Which of the random variables described below is/are discrete random variables?

The random variable X represents the number of heads when a coin is flipped 20 times.

The random variable Y represents the number of calls received by a car tow service in a year.

The random variable Z represents the weight of a randomly selected student.

Evaluate the expression.

9) \( \frac{11!}{6!} \)

10) \( \binom{10}{4} \)

Solve the problem.

11) A coin is biased so that the probability it will come up tails is 0.43. The coin is tossed three times. Considering a success to be tails, formulate the process of observing the outcome of the three tosses as a sequence of three Bernoulli trials. Complete the table below by showing each possible outcome together with its probability. Display the probabilities to three decimal places. List the outcomes in which exactly two of the three tosses are tails. Without using the binomial probability formula, find the probability that exactly two of the three tosses are tails.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>hhh</td>
<td>(0.57)(0.57)(0.57) = 0.185</td>
</tr>
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</table>

Find the indicated binomial probability. Be sure to follow the steps outlined below.

1. Identify a success
2. Determine the success probability, p.
3. Determine n, the number of trials.
4. Write the formula you will use to obtain the desired probability, for example \( P(X = 3) = \binom{5}{3}(0.3)^3(0.7)^2 = 0.1323. \)

12) What is the probability that 6 rolls of a fair die will show three fives?

13) In a certain college, 33% of the physics majors belong to ethnic minorities. If 10 students are selected at random from the physics majors, what is the probability that exactly 2 belong to an ethnic minority?

Find the indicated probability.

14) A test consists of 10 true/false questions. To pass the test a student must answer at least 6 questions correctly. If a student guesses on each question, what is the probability that the student will pass the test?

15) A machine has 7 identical components which function independently. The probability that a component will fail is 0.2. The machine will stop working if more than three components fail. Find the probability that the machine will be working.

16) In one city, the probability that a person will pass his or her driving test on the first attempt is 0.65. 11 people are selected at random from among those taking their driving test for the first time. What is the probability that among these 11 people, the number passing the test is between 2 and 4 inclusive?
17) In one city, the probability that a person will pass his or her driving test on the first attempt is 0.64. 11 people are selected at random from among those taking their driving test for the first time. What is the probability that among these 11 people, the number passing the test is between 2 and 4 inclusive?

Find the mean of the binomial random variable.

18) According to a college survey, 22% of all students work full time. Find the mean for the random variable X, the number of students who work full time in samples of size 16.

19) On a multiple choice test with 14 questions, each question has four possible answers, one of which is correct. For students who guess at all answers, find the mean for the random variable X, the number of correct answers.

Find the standard deviation of the binomial random variable.

20) According to a college survey, 22% of all students work full time. Find the standard deviation for the random variable X, the number of students who work full time in samples of size 16.

21) A die is rolled 22 times and the number of twos that come up is tallied. If this experiment is repeated many times, find the standard deviation for the random variable X, the number of twos.

Find the specified probability distribution of the binomial random variable.

22) A multiple choice test consists of four questions. Each question has five possible answers of which only one is correct. A student guesses on every question. Find the probability distribution of X, the number of questions she answers correctly.

Construct a probability histogram for the binomial random variable, X.

23) Two balls are drawn at random, with replacement, from a bag containing 4 red balls and 2 blue balls. X is the number of blue balls drawn.

Provide an appropriate response.

24) A bag contains 6 red marbles and 9 green marbles. Anne picks 3 marbles at random, with replacement, and observes the color of each marble. The number of green marbles, X, is a binomial random variable. If we let success correspond to getting a green marble, what is the success probability, p? What is the number of trials?
1) Answer: 0, 1, 2, 3, 4, 5
2) Answer: 0, 1, 2, 3, 4
3) Answer: \{X = 3\}
4) Answer: \{3 \leq Y < 5\}
5) Answer:
\[
\begin{array}{cc}
  x & P(X = x) \\
 0 & 0.167 \\
 1 & 0.278 \\
 2 & 0.222 \\
 3 & 0.167 \\
 4 & 0.111 \\
 5 & 0.056 \\
\end{array}
\]
6) Answer:

7) Answer: 0.55
8) Answer: X and Y
9) Answer: 55,440
10) Answer: 210
11) Answer: Each trial consists of observing whether the coin comes up heads or tails. There are two possible outcomes, heads or tails. The trials are independent. If we consider tails to be success, the success probability is \( p = 0.43 \).

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<tr>
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<tr>
<td>hht</td>
<td>((0.57)(0.57)(0.43) = 0.140)</td>
</tr>
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<td>hth</td>
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<tr>
<td>htt</td>
<td>((0.57)(0.43)(0.43) = 0.105)</td>
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</tr>
<tr>
<td>ttt</td>
<td>((0.43)(0.43)(0.43) = 0.080)</td>
</tr>
</tbody>
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htt, tht, tth; 0.316

12) Answer: A success is obtaining a five.

\[ p = \frac{1}{6} \]
\[ n = 6 \]
\[ P(X = 3) = \binom{6}{3} \left( \frac{1}{6} \right)^3 \left( \frac{5}{6} \right)^3 = 0.0536 \]
13) Answer: A success is that a person belongs to an ethnic minority.
   \[ p = 0.33 \]
   \[ n = 10 \]
   \[ P(X = 2) = \binom{10}{2} (0.33)^2 (0.67)^8 = 0.1990 \]

14) Answer: 0.377

15) Answer: 0.967

16) Answer: 0.0499

17) Answer: 0.0579

18) Answer: 3.52

19) Answer: 3.5

20) Answer: 1.66

21) Answer: 1.748

22) Answer:

<table>
<thead>
<tr>
<th>x</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.4096</td>
</tr>
<tr>
<td>1</td>
<td>0.4096</td>
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<tr>
<td>2</td>
<td>0.1536</td>
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<tr>
<td>3</td>
<td>0.0256</td>
</tr>
<tr>
<td>4</td>
<td>0.0016</td>
</tr>
</tbody>
</table>

23) Answer:

24) Answer: \( p = \frac{3}{5}, \ n = 3 \)