A hypothesis test is to be performed for a population mean with null hypothesis $H_0: \mu = \mu_0$. Find the required critical value(s).

1) $\alpha = 0.08$ for a two-tailed test.

2) $\alpha = 0.02$ for a left-tailed test.

3) A population is normally distributed with a standard deviation $\sigma = 89$. We wish to test the hypotheses

   $H_0: \mu = 25,534$  $H_1: \mu < 25,534$

   A 177-item sample has a mean $\bar{X} = 25,436.1$. Compute the value of the test statistic.

4) Jenny is conducting a hypothesis test concerning a population mean. The hypotheses are as follows.

   $H_0: \mu = 50$
   $H_a: \mu > 50$

   She selects a sample and finds that the sample mean is 54.2. She then does some calculations and is able to make the following statement:

   If $H_0$ were true, the chance that the sample mean would have come out as big (or bigger) than 54.2 is 0.3. Do you think that she should reject the null hypothesis? Why or why not?

5) The National Weather Service says that the mean daily high temperature for October in a large midwestern city is 56°F. A local weather service suspects that this value is not accurate and wants to perform a hypothesis test to determine whether the mean is actually lower than 56°F. A sample of mean daily high temperatures for October over the past 37 years yields $\bar{x} = 54°F$. Assume that the population standard deviation is 5.6°F. Perform the hypothesis test at the $\alpha = 0.01$ significance level.

6) A newspaper in a large midwestern city reported that the National Association of Realtors said that the mean home price last year was $116,800. The city housing department feels that this figure is too low. They randomly selected 66 home sales and obtained a sample mean price of $118,900. Assume that the population standard deviation is $3,700. Using a 5% level of significance, perform a hypothesis test to determine whether the population mean is higher than $116,800.

7) A brochure claims that the average maximum height a certain type of plant is 0.7 m. A gardener suspects that this estimate is not accurate locally due to soil conditions. A random sample of 41 mature plants is taken. The mean height of the plants in the sample is 0.65 m. Using a 1% level of significance, perform a hypothesis test to determine whether the population mean is different from 0.7 m. Assume that the population standard deviation is 0.2 m.

8) A hypothesis test for a population mean is to be performed at the 1% level of significance. True or false, if the null hypothesis is true, the probability that the test statistic will fall in the rejection region is 0.01?

9) A two-tailed hypothesis test for a population mean is to be performed at the 1% level of significance. The population standard deviation is known. True or false, the critical values are the two $z$-scores which divide the area under the standard normal curve into a middle 0.98 area and two outside areas of 0.01?

10) A hypothesis test for a population mean is to be performed. If the sample size is small (less than 15), under what conditions is it reasonable to use the $z$-test? If the sample size is moderate (between 15 and 30), under what conditions is it reasonable to use the $z$-test?
A one-sample z-test for a population mean is to be performed. Determine the P-value.

11) A right-tailed test:
   \[ z = 2.38 \]

12) A left-tailed test:
   \[ z = -2.65 \]

13) A two-tailed test:
   \[ z = 1.31 \]

14) In 1990, the average duration of long-distance telephone calls originating in one town was 9.4 minutes. A long-distance telephone company wants to perform a hypothesis test to determine whether the average duration of long-distance phone calls has changed from the 1990 mean of 9.4 minutes. The mean duration for a random sample of 50 calls originating in the town was 8.6 minutes. Do the data provide sufficient evidence to conclude that the mean call duration, \( \mu \), has changed from the 1990 mean of 9.4 minutes? Perform the appropriate hypothesis test using a significance level of 0.01. Assume that \( \sigma = 4.8 \) minutes.

15) A manufacturer considers his production process to be out of control when defects exceed 3%. In a random sample of 85 items, the defect rate is 5.9% but the manager claims that this is only a sample fluctuation and production is not really out of control. At the 0.01 level of significance, test the manager’s claim.

16) A poll of 1,068 adult Americans reveals that 52% of the voters surveyed prefer the Democratic candidate for the presidency. At the 0.05 level of significance, test the claim that more than half of all voters prefer the Democrat.

17) In a clinical study of an allergy drug, 108 of the 202 subjects reported experiencing significant relief from their symptoms. At the 0.01 significance level, test the claim that more than half of all those using the drug experience relief.

18) When performing a one-sample z-test for a population mean, what criterion do you use for rejecting the null hypothesis using the critical value approach? using the P-value approach? Assume that a right-tailed test is being performed.

19) A one-sample z-test for a population mean is to be performed. Let \( z_0 \) denote the observed value of the test statistic, \( z \). True or false, for a left-tailed test, the P-value is the area under the standard normal curve to the left of \( z_0 \)?

20) A one-sample z-test for a population mean is performed. Suppose that the P-value for the test is 0.04. For what significance levels (values of \( \alpha \)) can the null hypothesis be rejected?

21) A one-sample z-test for a population mean is to be performed. Why might it be more useful for those interpreting the results to know the P-value rather than simply whether or not the null hypothesis was rejected?

22) A manufacturer makes ball bearings that are supposed to have a mean weight of 30 g. A retailer suspects that the mean weight is actually less than 30 g. The mean weight for a random sample of 16 ball bearings is 28.5 g with a standard deviation of 4.1 g. At the 0.05 significance level, test the claim that the mean is less than 30 g.
23) A light-bulb manufacturer advertises that the average life for its light bulbs is 900 hours. A random sample of 15 of its light bulbs resulted in the following lives in hours.

<p>| | | | | | | |</p>
<table>
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<tr>
<td>995</td>
<td>590</td>
<td>510</td>
<td>539</td>
<td>739</td>
<td>917</td>
<td>571</td>
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<tr>
<td>916</td>
<td>728</td>
<td>664</td>
<td>693</td>
<td>708</td>
<td>887</td>
<td>849</td>
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At the 10% significance level, do the data provide evidence that the mean life for the company's light bulbs differs from the advertised mean?

24) Suppose that you wish to perform a hypothesis test for a population mean. Suppose that the population standard deviation is unknown, the population is skewed to the right, and the sample is large. Would you perform a z-test or a t-test? Why? Would the test be exact or approximate?

25) Suppose that you wish to perform a hypothesis test for a population mean. Suppose that the population standard deviation is unknown, the population is normally distributed, and the sample size is small. Would you perform a z-test or a t-test? Why? Would the test be exact or approximate?
Answer Key
Testname: CHAPTER7

1) ±1.75
2) -2.054
3) -14.63
4) Answers will vary. Possible answer. No, she should not reject the null hypothesis. If \( H_0 \) were true, the sample mean could easily be as big as 54.2 by chance. So the observed sample mean is consistent with the null hypothesis.
5) \( H_0 : \mu = 56^\circ \text{F} \)
   \( H_a : \mu < 56^\circ \text{F} \)
   Test statistic: \( z = -2.17 \).
   Critical value \( z = -2.33 \). Fail to reject \( H_0 : \mu = 56^\circ \text{F} \). There's not sufficient evidence to support the claim that the mean is less than 56°F.
6) \( H_0 : \mu = $116,800 \)
   \( H_a : \mu > $116,800 \)
   Test statistic: \( z = 4.61 \).
   Critical value: \( z = 1.645 \). Reject \( H_0 : \mu = $116,800 \). There is sufficient evidence to support the claim that the mean is higher than $116,800.
7) \( H_0 : \mu = 0.7 \text{ m} \)
   \( H_a : \mu \neq 0.7 \text{ m} \)
   Test statistic: \( z = -1.60 \).
   Critical values: \( z = \pm 2.575 \). Fail to reject \( H_0 : \mu = 0.7 \text{ m} \). There is not sufficient evidence to warrant rejection of the claim that the mean length is 0.7 m.
8) TRUE
9) FALSE
10) If the sample size is small, the z-test should only be used if the variable under consideration is normally distributed or close to being so. If the sample size is moderate, the z-test can be used unless the data contain outliers or the variable under consideration is far from being normally distributed.
11) 0.0087
12) 0.0040
13) 0.1902
14) \( H_0 : \mu = 9.4 \text{ minutes} \)
   \( H_a : \mu \neq 9.4 \text{ minutes} \)
   \( \alpha = 0.01 \)
   \( z = -1.18 \)
   \( P\text{-value} = 0.238 \)
   Do not reject \( H_0 \). At the 1% significance level, the data do not provide sufficient evidence to conclude that the mean call duration has changed from the 1990 mean of 9.4 minutes.
15) \( H_0 : p = 0.03 \)
   \( H_1 : p > 0.03 \)
   Test statistic: \( z = 1.57 \).
   P-value: \( p = 0.0582 \).
   Critical value: \( z = 2.33 \). Fail to reject null hypothesis. There is not sufficient evidence to warrant rejection of the manager's claim that production is not really out of control.
16) \( H_0 : p = 0.5 \)
   \( H_1 : p > 0.5 \)
   Test statistic: \( z = 1.29 \).
   P-value: \( p = 0.0993 \).
   Critical value: \( z = 1.645 \). Fail to reject null hypothesis. There is not sufficient evidence to support the claim that more than half of all voters prefer the Democrat.
17) \( H_0 : p = 0.5 \)
   \( H_1 : p > 0.5 \)
   Test statistic: \( z = 0.99 \).
   P-value: \( p = 0.1611 \).
   Critical values: \( z = 2.33 \). Fail to reject null hypothesis. There is not sufficient evidence to support the claim that more than half of all those using the drug experience relief.
18) Using the critical value approach, the null hypothesis is rejected if the test statistic is greater than the critical value. Using the P-value approach, the null hypothesis is rejected if the P-value is smaller than the significance level.
19) TRUE
20) For all values of \( \alpha \) greater than or equal to 0.04
21) The conclusion on whether or not to reject the null hypothesis may depend on the significance level selected and this is often selected fairly arbitrarily. The \( P \)-value gives the exact strength of the evidence against the null hypothesis.
22) \( H_0: \mu = 30 \text{ g.} \quad H_a: \mu < 30 \text{ g.} \)
   Test statistic: \( t = -1.46 \). Critical value: \( t = -1.753 \). Fail to reject \( H_0 \). There is not sufficient evidence to support the claim that the mean is less than 30 g.
23) \( H_0: \mu = 900 \text{ hours} \)
   \( H_a: \mu \neq 900 \text{ hours} \)
   Test statistic: \( t = -4.342 \). Critical values: \( t = \pm 1.761 \). Reject \( H_0: \mu = 900 \text{ hours} \). There is sufficient evidence to support the claim that the true mean life differs from the advertised mean.
24) The t-test is appropriate since the population standard deviation is unknown. The test would be approximate since the population is nonnormal.
25) The t-test is appropriate since the population standard deviation is unknown. The test would be exact since the population is normally distributed.