INTRODUCTION

Exponential behavior in electrical circuits is frequently referred to as "relaxation", particularly for circuits which make use of some exponential property for control of timing, eg. a "relaxation oscillator" makes use of the exponential charging of an RC circuit to control some repetitive process such as the flashing of warning lights at construction sites.

In an earlier experiment, the exponential behavior of a resistor-capacitor combination was investigated by making time measurements with a stopwatch, using fairly large values of both $R$ and $C$ in order to get a sufficiently slow variation. In this connection, it should be recalled that the time constant for an RC circuit is given by the product $RC$, and hence can be increased by increasing either of these quantities. In a resistance-inductance combination, on the other hand, the time constant is given by the ratio $L/R$. Attempts to obtain $L$ values large enough to allow direct measurement of exponential behavior by the stopwatch technique are therefore likely to fail because of the simultaneous increase in resistance that will occur as more windings are added to the inductor. (It is possible to obtain very long RL time constants by using superconducting technology to greatly reduce the resistance, but that is somewhat beyond the scope of this lab.)

In this experiment, the ability to display rapidly varying electrical signals on an oscilloscope will be used to allow the investigation of rapid exponential behavior of both RC and RL circuits. These procedures will then be extended to investigate the relaxation behavior of a series circuit containing all three components -- $R$, $L$, and $C$.

The graph in Figure 1 shows an exponential function starting at a vertical coordinate of 8 and decaying to zero. If we assume the horizontal axis to represent time, the half-life can be measured as the time required to fall from 8 to 4 on the vertical scale, which is 2 divisions on this axis. It is extremely important to note that it takes this same amount of time to get from any vertical value on the graph to a point which is 1/2 of the starting value. From 6 to 3, 4 to 2, 2 to 1, 1 to 1/2 ... all require 2 horizontal divisions. Thus, in order to measure half-life, any starting point will do, but it is absolutely necessary to be able to see enough of the curve to locate the asymptote in order to establish the "zero level".

The phrase "zero level" above is in quotes because of the possibility that the circuit under investigation might be asymptotically approaching some constant value other than zero. For example, it has been previously shown that when an RC circuit is initially connected to a battery, the charge on the capacitor will grow toward its equilibrium value with an exponential time constant identical to that which governs its decay, ie. the product $RC$. In this experiment both growing and decaying signals will be investigated, and it will be discovered that all such signals within the same circuit share the same half-life.
**THEORY**

For a complete theoretical treatment you should consult your textbook. The following is a brief summary of the essential points.

The \((V, I, t)\) and \((V, Q, t)\) dependencies for the three types of circuit components are shown below.

\[
V_R = RI = R \frac{dQ}{dt} \quad V_C = \frac{Q}{C} \quad I = C \frac{dV_C}{dt} \quad V_L = L \frac{dI}{dt} = L \frac{d^2Q}{dt^2}
\]

In most physics textbooks, the preferred variable is the set \((V, Q, t)\). Unfortunately, most of the rest of the world uses the set \((V, I, t)\) since \(V\) and \(I\) and \(t\) are readily measured with DMM's or oscilloscopes. In this lab we are going to investigate both an RC and an RL circuit driven by a square wave source.

**THE METHOD**

This discussion will be presented specifically for the RC circuit, and the variations that are needed for the RL case will be mentioned at the end. Rather than the batteries and switches customarily shown in textbook illustrations of RC circuits, a square wave generator will be used in order to accomplish the alternation between charging and discharging rapidly enough to provide a continuous signal for display on the oscilloscope. A square wave "instantaneously" switches back and forth, at a steady frequency, between a plus voltage and zero voltage (or ground). When connected in series with an RC pair, this changing voltage has the effect of trying to instantaneously reverse the charge on the capacitor at this same frequency but, since the charge can move only as fast as allowed by the resistor, it can't keep up with the square wave. The voltage across the capacitor will therefore resemble Figure 2, showing exponential behavior first in one direction and then in the other.

**Figure 2**

1. RC Circuit Theory

   (A) If \(V_S = 0\) (square wave at zero value) then \(dV_C/dt + V/R_C = 0\) and \(V_C(t) = V_o e^{-t/RC}\)

   Remember that the initial condition is \(V_C(t=0) = V_0\), due to the previous charge cycle.

   (B) If \(V_S = V_o\) (square wave at \(V_o\) value) then \(dV_C/dt + V/R_C = V_o/R_C\) and

   \[V_C(t) = V_o \left[1-e^{-t/RC}\right]\]

   Recall that the initial condition for the capacitor voltage is \(V_C(t = 0) = 0\), due to the previous discharge cycle.
(C) Sketch of both solutions

2. RL Circuit Theory

\[
\begin{align*}
\text{KVL} & \rightarrow -V_s + V_L + V_R = 0 \\
V_L + V_R & = V_s \\
L(dI/dt) + IR & = V_s \\
\text{so } dI/dt + (R/L)I & = V_s/L
\end{align*}
\]

(A) When \( V_s = 0 \) then \( I(t) = I_0 e^{-(R/L)t} \)

(B) When \( V_s = V_o \) then \( I(t) = I_0 [1 - e^{-(R/L)t}] \)

But with an oscilloscope you measure \( V(t) \), not \( I(t) \). So multiply both sides of the \( I(t) \) equations by \( R \) and:

\[
\begin{align*}
V_R(t) & = V_o e^{-(R/L)t} \text{ (discharge)} \\
V_R(t) & = V_o [1 - e^{-(R/L)t}] \text{ (charge)}
\end{align*}
\]

(C) Sketch of both solutions

Just like part (C) in section 1 above, except you are looking at \( V_s(t) \) and \( V_R(t) \).

EQUIPMENT

... Scope Probes

Special purpose cables, called probes, are used when connecting an oscilloscope into an operating circuit. These cables have the obvious function of providing clip-on connections small enough to hook onto the component under study without interfering with other circuit elements. They also do a few other things that are not so immediately evident.

As with any measuring device, one of the principal concerns in scope measurements is to avoid changing the signal being measured in the act of measuring it. Since scopes are frequently used to examine rapidly changing signals of complicated form, some care must be taken so that the resistive and reactive properties of the connecting cables do not distort the shape of the signal. A good probe contains compensating circuitry to exactly cancel the effects of that particular length of that particular type of co-ax cable. This circuitry may be located near the tip of the probe or, for smaller diameter probe cables, in a small box at the scope end. An additional function of probe compensation is to avoid reflections of the signal which could produce "ghosts" on the display.
Whenever a probe is initially connected to a scope, it is important to check this compensation. Most scopes have a small test point on the front panel at which a very precise square wave is provided for probe calibration. Touch the probe tip to this point and get a display of this signal. Then adjust the probe as needed to make the corners of the wave sharp and square. The probes are adjusted by a small screwdriver through the small hole on the box at the scope end of the cable. **Normally you will not need to adjust the probe with the o’scope and probes currently in use.**

It is also very important to note that all of the probes available in the physics lab **attenuate** (reduce) the signal being measured by a factor of ten. Thus, whenever a probe is in use, the Volts/div knobs on the scope should be read at the index that is marked "**x10 probe**".

**... Ground connections**

Remember that the outer shells of all the co-ax (BNC) connectors on the scope are internally connected together, and are connected to ground through the third wire in the power cable. It is therefore very important to take care that only one point gets grounded in the circuit being measured. Any time that two separate points get grounded everything between these points is "shorted" through ground ... just as if a wire were connected across them. Thus there are two rules to be observed:

1. Only one probe should ever have a ground "pigtail" installed at the tip.
2. This ground should always be connected to the circuit at the low or ground side of the generator.

**GENERAL PROCEDURE for Half-Life Measurements**

A. Get a stable scope display of the desired signal. (Hint: make sure the trigger source is set on INT (internal trigger) and is using the signal from the Channel being measured)

B. Adjust the sweep rate (time/div) so that several cycles are seen, and adjust the vertical amp (Volts/div) to approximately fill the screen. Your display should now look more or less like Figure 2 (either A or B depending upon what signal is being measured.)

C. If your display does not clearly show the asymptotic part of the exponential, ie. if it looks more triangular than exponential, then your square wave frequency is set too fast. This simply shows that the polarity (square wave) changes are taking place much too quickly for the circuit to keep up. Reduce the generator frequency until the horizontal asymptote can be seen. Likewise, if the asymptote seems to go on forever the generator speed should be increased in order to get about 2 charge/discharge cycles on the screen.

D. Now use your head (think!) and adjust the display (horizontal and vertical) to give you a charge or discharge pattern that fills the o’scope screen as completely as possible. As per the experiment instructions on the next page, make sure that you are using both Channel 1 and Channel 2. You should be able to make a reasonably accurate measurement of the ½ life for either the charge curve or the discharge curve. If you are very clever, you will soon figure out a way of adjusting the time base (sec/cm – horizontal) to stretch the display out, thereby enabling you to make a much more accurate measurement of the ½ life.
THE EXPERIMENT

Throughout these experiments, it must be kept in mind that $R$ represents the total series resistance in the circuit. This includes the internal resistances of the generator, of the inductor, and of all connecting wires, as well as any component resistor(s) that may be installed. The connecting wires make a very tiny contribution to the total and may safely be ignored. The inductor, on the other hand, contains many turns of very fine wire, and thus has a substantial resistance. Measure it!! (Hint:...ohmmeter).

There are several techniques for determining the internal resistance of a generator, all closely related to those used in the earlier experiment on the Load-Line of a battery. The generators available in the physics lab have resistances of 50 ohms on all scales, Sine or Square.

1. Connect the RC circuit as shown below, with initial values of $R = 5.6\ \text{k}\Omega$ and $C = 0.1\text{µF}$. Measure the half-life for $V_C$ and compare with the expected value.

Your oscilloscope has 2 input channels, 1 and 2. In normal circumstances, you should always use both, one for the input signal to your circuit and the other for the output signal. That way you can see the input and the output and you can easily compare them by having both on the screen at the same time.

![Diagram of RC circuit]

Generally you should trigger the oscilloscope on CH 1 since the input signal is more likely to be consistent and easy to trigger with. Notice that the signal generator and both channels of the O’scope must have a common ground since they are connected through the shield of the co-axial cable and/or the 3rd wire on the power plug.

2. Repeat for enough different RC combinations to convince yourself that you understand the method.

3. Replace the resistor with a variable resistor, and describe the behavior of the exponential as the resistance is varied. Does this make sense to you? Can you use a half-life measurement to determine the resistance of a particular setting on the variable resistor? TRY IT! Check your result by using the ohmmeter.

4. Reconnect the circuit so you can measure the half-life for $V_R$ and confirm that you get the same value. This requires some thought about the ground location. (Hint … Look at the RL circuit in section 5 below.)

Hints: You might naively think that you simply connect CH 2 across R to measure $V_R$. But there are problems! If you connect CH 2 ground to the left (+) side of R, you are grounding the + side of the signal generator, thereby possibly damaging it. If you connect CH 2 ground to the right (-) side of R, then the capacitor is grounded top and bottom (shorted) so it is effectively removed from the circuit.

So what can you do? - Think about interchanging R and C in the circuit.

Then go ahead and make your measurements for $V_R$. (Of course, you can’t make simultaneous measurements of $V_C$ now.)
5. Now examine at least one RL circuit (R = 1kΩ and L ~20mH are reasonable values). Compare the measured half-life with the theoretically expected value.

6. Examine the shapes of the exponential voltage signals across both the resistor and the inductor. Give a qualitative explanation for these shapes in a manner similar to that presented above (in the METHOD section) for the RC circuit. Do these two voltages in fact add up to a square wave as demanded by Kirchoff’s rule? Explain your answer!

7. There is a clever way of avoiding the ground problem for the RC and RL circuits which allows you to look with the o’scope at both VR and VC at the same time (or VR and VL). *

Build a circuit as shown. Make sure that both R’s are nearly identical in value and the same for the capacitors or inductors.

Connect CH1 across VC or VL and connect CH2 across VR. You can now study VR and VC or VL at the same time. Sketch the two responses (VR and VC, or VR and VL). [Make sure that you are using the same vertical amplitude (V/cm) setting for both channels.]

Now find the setting (switch) on the o’scope that allows you to see CH 1 plus CH2. (Add CH1 and CH 2.) If you are set up properly, you should see a square wave. Why? Explain! Does this observation agree with your sketches of VR and VC or VR and VL?

*A clever student suggested this some years ago.